MATH 3063 ABSTRACT ALGEBRA — PROF. PAUL BAILEY — Project 1 — January 23, 2009 — Name:

Due Friday, January 30, 2009.

Copy the statement of the problem on a piece of $8\frac{1}{2} \times 11$ piece of blank computer paper, and write the solution underneath. Write neatly. Mathematics should always be written in grammatically correct English, in complete sentences.

Problem 1. Use induction to prove that, for all $n \in \mathbb{N}$,

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$

Problem 2. Let $m = 71$ and $n = 528$. Find $x, y, d \in \mathbb{Z}$ such that $mx + ny = d$ and $d = \gcd(m, n)$.

Problem 3. Let $a, b, c \in \mathbb{Z}$ be positive integers. Show that

(a) $a \mid a$;

(b) $a \mid b$ and $b \mid a$ implies $a = b$;

(c) $a \mid b$ and $b \mid c$ implies $a \mid c$.

Problem 4. Let $a, b, c \in \mathbb{Z}$ be positive integers.
Show that $\gcd(a, bc) = 1$ if and only if $\gcd(a, b) = 1$ and $\gcd(a, c) = 1$.

Problem 5. Find the additive order of $\overline{6}$, $\overline{11}$, $\overline{18}$, and $\overline{28}$ in $\mathbb{Z}_{36}$.

Problem 6. Find the multiplicative order of $\overline{10}$ in $\mathbb{Z}_{21}^*$.

Problem 7. Solve the equation $\overline{17}x = \overline{23}$ in $\mathbb{Z}_{71}$.

Problem 8. Solve the equation $x^2 - \overline{5}x - \overline{2} = \overline{0}$ in $\mathbb{Z}_{11}$.